Roll-Pitch Gyro Drift Compensation

William Premerlani
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Problem

We wish to use an accelerometer and GPS to detect gyro roll-pitch drift in an IMU, without any information about orientation of the aircraft with respect to air speed vector.

Background

During the development of the theoretical foundations of MatrixPilot, the conventional wisdom of the DIY UAV community was that the best way to account for acceleration in roll-pitch drift compensation is to compute the acceleration in the body frame, using GPS acceleration to determine forward acceleration, and the cross product of the aircraft velocity vector with gyro rotation to compute cross acceleration. In order for this technique to be accurate, the angles of attack and side slip must be known, estimated, or assumed. In the case of fixed wing aircraft, this introduces errors during high acceleration. In the case multicopters, this introduces very large errors, because a multicopter does not naturally align itself with the direction of flight. So, a method that does not require attack and side slip angles would be more accurate.

Solution

The solution is trivial if you use gravity minus acceleration as the reference vector, rather than gravity. In the body (aircraft) frame of reference, the vectors reported by the accelerometer are the differences between the gravity and acceleration vectors, as seen in the body frame of reference:

$$\begin{aligned}
&\mathbf{A_b}(t) = \mathbf{g_b}(t) - \mathbf{a_b}(t) \\
&\mathbf{g_b}(t) = \text{gravity, as seen in body frame} \\
&\mathbf{a_b}(t) = \text{acceleration, as seen in body frame} \\
&\mathbf{A_b}(t) = \text{output of accelerometer}
\end{aligned} \qquad \text{Equation 1}$$

Our estimate of the rotation matrix provides a connection between body and earth frame of reference:

$$\begin{aligned}
&\hat{\mathbf{R}}(t) \cdot \mathbf{A_b}(t) = \mathbf{g_e} - \mathbf{a_e}(t) \\
&\mathbf{g_e} = \text{gravity, as seen in earth frame} \\
&\mathbf{a_e}(t) = \text{acceleration, as seen in earth frame} \\
&\hat{\mathbf{R}}(t) = \text{our estimate of the rotation matrix}
\end{aligned} \qquad \text{Equation 2}$$

If our estimate of the rotation matrix were correct, equation 2 would be an identity. However, because of gyro drift, equation 2 will not be exactly satisfied. In particular, the cross product of the vectors on the left and right hand side of Equation 2 is an indication of the drift error. To improve accuracy, it is a good idea to integrate both sides of the equation between GPS reports. That way we can use velocity information instead of acceleration:

$$\int_{t1}^{t2} \breve{\mathbf{R}}(\tau)\cdot \mathbf{A_b}(\tau)\cdot d\tau = (t2-t1)\cdot \mathbf{g_e} - (\mathbf{V_e}(t2)-\mathbf{V_e}(t1)) \qquad \text{Equation 3}$$

Use equation 3 on a regular basis (such as synchronized with GPS messages) to estimate drift. The left side is computed as the sum of multiplications of the rotation matrix with the accelerometer output. The first term on the right hand side is the time interval times the known, constant gravity vector. The second term is the difference in velocity vectors reported by the GPS.

Take the cross product of the left hand side of equation 3 with the right hand side. This is an indication of the drift rotation. Note that this method is capable of detecting yaw drift as well as roll-pitch, if the aircraft is accelerating laterally. So, it can be integrated with magnetometer yaw drift compensation. Also note that no assumptions have been made about the orientation of the aircraft.

So, the error rotation vector is given by:

$$\mathbf{error}_{earth}(t2) = \left(\int_{t1}^{t2} \breve{\mathbf{R}}(\tau)\cdot \mathbf{A_b}(\tau)\cdot d\tau\right) \times \left((t2-t1)\cdot \mathbf{g_e} - (\mathbf{V_e}(t2)-\mathbf{V_e}(t1))\right)$$

Equation 4

The error computed by equation 4 is quadratic in gravity minus acceleration, and is also quadratic in the time between GPS reports. It would be better to have an expression that is linear in gravity minus acceleration, and independent of the time interval:

$$\mathbf{error}_{earth}(t2) = \frac{\left[\frac{1}{t2-t1}\int_{t1}^{t2} \breve{\mathbf{R}}(\tau)\cdot \mathbf{A_b}(\tau)\cdot d\tau\right] \times \left[\mathbf{g_e} - \frac{(\mathbf{V_e}(t2)-\mathbf{V_e}(t1))}{t2-t1}\right]}{\left|\mathbf{g_e} - \frac{(\mathbf{V_e}(t2)-\mathbf{V_e}(t1))}{t2-t1}\right|} \qquad \text{Equation 5}$$

The denominator in equation 5 is the absolute value of gravity minus the average acceleration. Basically, it normalizes the gravity-acceleration reference vector.

Finally, transform the error in the body frame of reference. It can then be used directly as an input to the drift PI feedback controller:

$$\mathbf{error}_{body} = \breve{\mathbf{R}}^{\mathbf{T}}(t2)\cdot \mathbf{error}_{earth}(t2) \qquad \text{Equation 6}$$

Appendix - Advantage of working in earth frame of reference

It is sometimes suggested that the computations can be done equally as well in the body frame of reference. If the reporting rate of the GPS is the same as the gyros and acclerometers, that would be true. But typically the gyros and accelerometers are updated much more frequently than the reporting rate of GPS. In MatrixPilot, gyros and acclerometers are measured and integrated 8000 times per second. The fastest reporting rate of GPS is around 10 Hz, and some (such as the EM406) report once per second. In the case of high sampling rates for gyros and accelerometers, much higher accuracy can be obtained in the earth frame of reference. The reason why can be discovered by trying to develop an equation similar to equation 3, except in the body frame.

First, we note that equation 3 is exact. The left side involves an integral. By matching the time window of the integral to the reporting rate of the GPS, an equation is produced that exactly matches the integral of the accelerometer reading in the earth frame of reference to the change in velocity reported by the GPS in the earth frame of reference.

So, lets see what sort of equation can be developed in the body frame. Start with equation 2, except express it in the body frame:

$$\mathbf{A_b}(t) = \breve{\mathbf{R}}^{\mathbf{T}}(t) \cdot \left[\mathbf{g_e} - \mathbf{a_e}(t)\right] \qquad \text{Equation 7}$$

The problem is, we do not have the acceleration in the earth frame at the same time that we have the other quantities in equation 7. We would be very tempted to use the average acceleration in the earth frame, but that would only be an approximation to what we really need, and we would not get the sort of exact answer that equation 3 produces. We can see where the problem lies by taking the integral of equation 7 between GPS reports:

$$\begin{aligned}
\int_{t1}^{t2} \mathbf{A_b}(t) \cdot dt &= \int_{t1}^{t2} \breve{\mathbf{R}}^{\mathbf{T}}(t) \cdot \left[\mathbf{g_e} - \mathbf{a_e}(t)\right] \cdot dt \\
\int_{t1}^{t2} \mathbf{A_b}(t) \cdot dt &= \int_{t1}^{t2} \breve{\mathbf{R}}^{\mathbf{T}}(t) \cdot \mathbf{g_e} \cdot dt - \int_{t1}^{t2} \breve{\mathbf{R}}^{\mathbf{T}}(t) \cdot \mathbf{a_e}(t) \cdot dt
\end{aligned} \qquad \text{Equation 8}$$

It would be easy enough to compute the left side of equation 8, and the first term on the right side. However, the second term on the right side gives us pause. We can not get the instantaneous acceleration from the GPS, all we can get is the average value. And that is not the same thing as the second term in equation 8, especially when the plane is turning. So it is much better to use equation 3.